PG – 388

III Semester M.Sc. Degree Examination, December 2016
(CBCS)
MATHEMATICS
M302T : Mathematical Methods

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Solve the following integral equation :

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$$

b) Arrive at the integral equation corresponding to the IVP : $y'' + y = e^x$; $y(0) = y'(0) = 0$. **(7+7)**

2. a) Solve by Laplace transform the integral equation

$$y(x) = \sin x + 2\int_0^x \cos(x-t)y(t)dt.$$

b) Find the Neumann series solution and resolvent kernel for the volterra integral equation.

$$y(x) = (1+x) + \lambda \int_a^x (x-t)y(t)dt.$$ **(6+8)**

3. a) Obtain for small x the asymptotic form of $I(x) = \int_x^{\infty} e^{-t^4} dt$.

b) Using the asymptotic series of the integral : $I(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt$, for $x \to \infty$ examine whether the integral is convergent or not. **(7+7)**

P.T.O.

4. a) Find the asymptotic expansion of

$$I(x) = \int_{-\pi/2}^{\pi/2} e^{x\cos t}\, dt \text{ as } x \to \infty$$

using Laplace method.

b) Evaluate using Watson's lemma :

$$I(x) = \int_{0}^{\pi/2} e^{-x\sin t}\, dt\ .$$

c) Find the leading order term of

$$I(x) = \int_{0}^{\infty} \cos(xt^3 - t)\, dt \text{ by}$$

stationary phase method. **(5+5+4)**

5. a) Derive Adam's predictor-corrector method of fourth-order for solving

$$\frac{dy}{dx} = f(x, y, (x))\ ;\ y(x_0) = y_0.$$

b) Discuss about absolute or relative stability of Runge-Kutta method of first order. **(10+4)**

6. a) Solve by finite-difference method

$$\frac{d^2y}{dx^2} + xy = 0,\ y(0) = 0 \text{ and } y'(1) = 1.$$

Choose $\Delta x = \frac{1}{3}$.

b) Solve the one-dimensional ware equation $U_{tt} = U_{xx}$, $0 \le x \le 1$, $t \ge 0$

Subject to $\left.\begin{aligned} u(x,0) &= \sin(\pi x) \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}\right\},\ 0 \le x \le 1,$

$\left.\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= 0 \end{aligned}\right\},\ t \ge 0,$ by using explicit finite difference method. Choose $\Delta x = \frac{1}{4}$ and an acceptable value for Δt. Obtain the solution at the second time-level. **(6+8)**

7. Discuss the stabilities of the Schmidt explicit method and Crank-Nicolson implicit method applied to the one-dimensional heat equation. **14**

8. Solve by finite difference method the following BVP :

   $U_{xx} + U_{yy} = 0,$

   Subject to the boundary conditions

   $u(x, 0) = 3x, 0 \leq x \leq 3.$

   $u(3, y) = 9 - 3y, 0 \leq y \leq 3,$

   $u(x, y) = x(1 - x)$ on $y = 3x, 0 \leq x \leq 1$

   $u(x, 3) = 0, 0 \leq x \leq 3.$

   Choose $\Delta x = \Delta y = 1$ and solve the system of equation by 4 iterations of the Gauss-Seidel method. **14**